this instability led to extinguishment. It is interesting to note that this type of dynamic instability has been treated by Akiba and Tanno[8] and their linearized theory predicts that the slope of the L^* vs P_e correlation should be $(-2n)$ where n is the pressure exponent of the propellant. For JPL 534, $n = 0.86$ and therefore our data on this propellant agree quite well with their linearized theory.

At the present time there is no theoretical treatment for propellants containing solid products. However, one would expect these propellants to exhibit less pressure sensitivity because the presence of solids in the burning zone should lessen the effect of pressure level on the characteristic overshoot time. This was observed in the aluminized propellant (Fig. 3).

Conclusions

This preliminary work does not answer all the questions concerning low pressure extinction, but it does indicate that a few experimental vacuum firings using regressive grains can be used to determine the low pressure limit for any particular propellant formulation. This work also indicates that the theory of the phenomena needs further study, particularly for the case of propellants that yield a sizeable concentration of solid products when they burn.

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Simulation of Solid Propellant Exhaust Products with a Hybrid Rocket Motor

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SEVERAL major disadvantages have always been associated with the use of solid propellant rocket motors for certain types of test programs. These difficulties are summarized in the following paragraphs:

1) The cost of tooling is often prohibitive. Tooling costs are often encountered for each individual test, since test parameters cannot always be met with existing tooling. Propellant preparation and fabrication is expensive, due to the hazards involved. Remote handling must be provided, which results in high equipment costs and reduced efficiency.

2) There is a lack of test control. Once ignited, it is difficult to exert control over the solid propellant firing. In testing developmental rocket motor components, it is often desirable to test progressively (i.e., short firings of increasing duration). In such a program, it is not unlikely that failures will occur. In this event, the firing should be terminated immediately so that the nature of the failure may be determined before the unit is destroyed. Thus, it would be possible to repair and modify the unit and repeat the test. To accomplish this at present, short solid propellant firings must be scheduled, making it necessary to use a new motor each time. In the over-all picture, these short firings cost almost as much as longer ones. Progressive type testing, therefore, becomes prohibitive in cost and time.

3) Once ignited, the propellant is difficult to extinguish, and once it is extinguished, the grain cannot be reused. With regard to test termination, it is possible to install a blowout port or a similar device. However, once the propellant is ignited, it can be extinguished only through proper loss of pressure at a given rate. If a test is terminated, the entire propellant is lost along with the associated costs.

To avoid these difficulties inherent in solid propellant testing, attempts have been made to simulate the solid propellant exhaust. These have primarily consisted of adding dry powders to a hydrogen-oxygen rocket motor. Control of the flow rate of these powders, however, has proved to be difficult, and these simulation methods have not been completely successful.

In recognition of these problems, a hybrid rocket motor was conceived which would circumvent all the disadvantages just listed, while providing a reasonable simulation of the solid propellant exhaust products. Briefly, the solid propellant constituents, without oxidizer, are formed by conventional molding techniques. This composition constitutes the "grain." The oxidizer is then supplied externally through an injector. With removal of the oxidizer, such a grain can be molded safely in standard fabrication areas with minimum cost in tooling. A wide range of grain binder materials can be used, since the hybrid concept eliminates the usual stringent physical property requirements. Unbonded fuel grain segments, fabricated by a room temperature compaction molding process using temporary tooling, would be possible.

The simplest solid propellant formulation to simulate with the hybrid motor concept is an ammonium perchlorate-aluminum-binder system. The basic constituents of such a system may be represented chemically as,

$$NH_4ClO_4 + x(Al) + y(C_aH_b)$$

where C_aH_b is the organic binder.

It should be noted that by removing the oxygen from the ammonium perchlorate, it becomes ammonium chloride, a stable salt. Ammonium chloride, aluminum powder, and organic binder can be efficiently molded into a grain by using conventional techniques. The oxygen can then be supplied externally, resulting in the following chemical form:

$$2O_2 + NH_4Cl + x(Al) + y(C_xH_y)$$

Chemically, the only difference in the two chemical systems just depicted is the heat of formation of ammonium perchlorate, which is 3300 cal/mol. This results in a slight lowering of the combustion temperature in the simulated system. An estimate of this temperature difference may be obtained by assuming that this heat is removed after combustion, as follows:

$$3300\,\frac{\text{cal}}{\text{mol}} \times \frac{\text{mol}}{117\text{ g}} = 28.2\,\frac{\text{cal}}{\text{g NH}_4\text{ClO}_4}$$

Assume the propellant composition to be simulated containg 60% NH_4ClO_4. For every gram of NH_4ClO_4, there are 1.67 s

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Table 1 Experimental data on hybrid rocket motor

Aluminum content, %	Pyrometer temperature reading, °F	Maximum chamber pressure, psia
18	4880	395
18	4840	370
18	5110	400
18	4730	365
18	4890	300
40	5080	350

of exhaust products. An average specific heat for the exhaust products would be, conservatively, 0.25 (cal/g °C). For 1.0 g of NH_4ClO_4, therefore,

$$\Delta T = (-Q/mCp)$$

where

ΔT = temperature change, C°
Q = quantity of heat in calories
m = mass in grams
C_p = specific heat, cal/g °C

$$\Delta T = (-28.2/1.67 \times 0.25) = -67.5°\text{C}$$
$$\Delta T = -122°\text{F}$$

This temperature change is not considered significant. It is acknowledged, however, that such a mixing of the solid fuel and gaseous oxidizer will never be exactly the same as in the true solid propellant, and good performance will depend on proper oxidizer injector and motor design.

Experimental Studies

Fuel grains were molded and machined in the form of a hollow cylinder, 1.78 in. o.d. × 0.5 in. i.d. × 1.65 in. These were designed to fit within a small water-cooled rocket motor. An injector directed oxygen to the fuel grain. Several binder systems and binder additives were used in the grain formulations. A hydrogen-oxygen pilot was used to ignite the grain. After 1 to 2 sec of pilot ignition, the main oxygen valve was opened. All grains ignited without difficulty, and burned smoothly. The chamber pressure rose rapidly and reached a steady state value, in most cases, before the grains were consumed. Chamber pressures as high as 400 psi were recorded. Firing durations were of the order of 4 sec.

A refractory material was placed downstream of the nozzle exit, and an optical pyrometer was sighted on its surface to obtain an estimate of the combustion temperature. The data from these runs are represented in Table 1. Generally, the data indicate that the combustion temperature was in the vicinity of 5000°F. This temperature is only slightly below the 5300°–6400°F range for ammonium perchlorate-aluminum systems.

Comments

Covariance Matrix Approximation

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The determinant of the covariance matrix of a trivariate normal distribution is a figure of merit which relates to error volumes in tracking and prediction problems. In computing this determinant, neglect of the covariance elements leads to pessimistic estimates of system accuracy, hence is a safe approximation for preliminary analysis. This property is verified, and a comparison of error volume results obtained with and without inclusion of covariance elements is given for different degrees of correlation between errors.

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The purpose of this note is to call attention to a useful approximation in error analyses involving trivariate normal distributions. By way of background, such a distribution in the error coordinates x,y,z, all assumed to be normalized to zero means, has as density function

$$p(x,y,z) = (2\pi)^{-3/2}|M|^{-1/2} \exp\{-\tfrac{1}{2}[xyz]M^{-1}[xyz]^T\} \quad (1)$$

where -1 and T indicate matrix inverse and transpose. M is the symmetric covariance matrix of the distribution, defined by

$$M = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad (2)$$

where σ_{xx} is the variance of x error, $\sigma_{xy} \equiv \sigma_{yx}$ is the covariance of x and y errors, etc. Taking the errors x, y, z to be referred to rectangular coordinate axes, a rotation of these axes can always be accomplished by an orthogonal transformation such that, in the new axis system (denoted by primes), the errors x',y',z' are independent. In the primed system, the covariance matrix M' has as elements of its principal diagonal the variances $\sigma_{x'x'},\sigma_{y'y'},\sigma_{z'z'}$ of x',y',z' errors, with all off-diagonal elements being zero. The determinant $|M'|$ has the property that $4/3\pi|M'|^{1/2}$ is equal to the volume of an ellipsoid aligned with the primed axes and having semi-axes $\sigma_{x'} \equiv (\sigma_{x'x'})^{1/2}$, $\sigma_{y'},\sigma_{z'}$. Corresponding to this error volume is a probability level of about $P = 0.2$, and other levels can be associated with similar ellipsoids defined by semi-axes $k\sigma_{x'},k\sigma_{y'},k\sigma_{z'}$, where k is any positive constant.

In many vehicle tracking and prediction problems where x, y, z are rectangular position coordinate errors, the sizes of the preceding error volumes (for different k) are taken as measures of system accuracy, since it is within such volumes, centered about the measured or computed position of the vehicle, that it may be expected to lie with different probability levels P. Thus, $|M'|$ is a fundamental figure of merit. To calculate $|M'|$, it is not necessary to transform the original distribution to the primed system in which the errors are independent, because from the orthogonality of the required transformation it follows that $|M| = |M'|$. That is, it is sufficient to work with the determinant of the covariance matrix of the original distribution. Even allowing for this property, however, it is still inconvenient in preliminary analyses to have to deal with the covariance elements. The question then naturally arises as to the type and size of error introduced by approximating $|M|$ as merely the product of the principal diagonal variance elements only, i.e., by assuming all covariances to be zero.

The point of the present note is simply this: the foregoing approximation is a pessimistic or "safe" one, in that the determinant so obtained is always greater than (or equal to) the true value of $|M|$ with covariance elements included. Relating $|M|$ to error volumes, if the accuracy requirements for a particular system are satisfied for $|M|$ approximated in this way, a more thorough analysis incorporating covariance is obviated, for the results can only be more (or equally) favorable in terms of system performance exceeding requirements.

To prove the inequality $\sigma_{xx}\sigma_{yy}\sigma_{zz} \geq |M|$, introduce correlation coefficients $\rho_{xy} \equiv \sigma_{xy}/(\sigma_{xx}\sigma_{yy})^{1/2}$ and ρ_{xz}, ρ_{yz}, similarly defined. Then, by expansion of $|M|$,

$$|M| = \sigma_{xx}\sigma_{yy}\sigma_{zz}\,[1 - (\rho_{xy}^2 + \rho_{xz}^2 + \rho_{yz}^2 - 2\rho_{xy}\rho_{xz}\rho_{yz})] \quad (3)$$